11. Homework Assignment

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Consider the vector field

$$\dot{x} \;=\; f(x) \;\in\; \mathbb{R}^N,$$

with $f(0) = 0$. Assume that the linearization $Df(0)$ possesses an algebraically simple eigenvalue 0 and all other eigenvalues have nonzero real part. Can there exist nonstationary periodic orbits in arbitrarily small neighborhoods of $x = 0$?

Problem 2: Consider a vector field $f : \mathbb{R}^N \to \mathbb{R}^N$ of the flow φ_t with equilibrium $x_0 = 0$. Let $R : \mathbb{R}^N \to \mathbb{R}^N$ be a linear involution ($R^2 = \mathrm{id}$). Assume that the vector field f is equivariant under the symmetry R, that is

$$f \circ R \;=\; R \circ f.$$

(i) Let the assumptions of the theorem on the existence of a *global* center manifold in $x_0 = 0$ be satisfied. Prove that the center manifold $W^c(x_0)$ is symmetric with respect to R, that is
$$R(W^c) \;=\; W^c.$$

(ii) Let the assumptions of the theorem on the existence of a *local* center manifold in $x_0 = 0$ be satisfied. Is every local center manifold symmetric with respect to R?

(iii) Prove that there exists a symmetric local center manifold $W^c_{\mathrm{loc}}(x_0)$, i.e.

$$R(W^c_{\mathrm{loc}}) \;=\; W^c_{\mathrm{loc}}.$$

Problem 3: As in class consider the homogeneous polynomials:

$$H_2(\mathbb{R}^2) \;=\; \operatorname{span}\left\{ \begin{pmatrix} x^2 \\ 0 \end{pmatrix}, \begin{pmatrix} xy \\ 0 \end{pmatrix}, \begin{pmatrix} y^2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ x^2 \end{pmatrix}, \begin{pmatrix} 0 \\ xy \end{pmatrix}, \begin{pmatrix} 0 \\ y^2 \end{pmatrix} \right\}.$$

Determine the image of the map $\operatorname{ad} A(H_2(\mathbb{R}^2))$ for the matrix A:

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Problem 4: Consider the linear system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\lambda_1 x \\ -\lambda_2 y \end{pmatrix}$$

with $0 < \lambda_1 < \lambda_2$. Depending on λ_1, λ_2, determine all flow-invariant one-dimensional manifolds which are tangent the x-axis and determine their smoothness class $\mathcal{C}^k$.